STABILITY OF A VISCOUS COMPRESSIBLE SHEAR LAYER

WITH A TEMPERATURE GRADIENT

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We consider the stability to small perturbations of a free shear layer formed by mixing two parallel streams of a viscous compressible gas. The temperatures T_1 and T_2 of the two streams may be different. For example, large temperature differences are typical for the boundary layer of a jet flowing out of the nozzle of an airplane or rocket motor. The stability of flows of this kind is of interest in connection with turbulization and noise-generation processes at high velocities [1-3].

The present paper is a continuation of [4], where the stability of a compressible shear layer was studied for $\kappa = 1$ ($\kappa \equiv T_2/T_1$). As in [4], the stability problem is solved using the linearized Navier–Stokes equations for a compressible gas and the orthogonalization method [5]. Solutions are calculated over a wide range of the parameters: Reynolds number ($0 < \mathrm{Re} \le 10^3$), Mach number ($0 \le M \le 2$) and the temperature difference ($0.2 \le \kappa \le 5$). It is found that for different values of M and κ three discrete perturbation modes can be unstable. One of these modes propagates with a subsonic phase velocity, while the other two are "supersonic." The neutral stability curves and the growth constants are determined. It is shown that perturbations of the continuous spectrum are damped for finite values of Re and all values of κ. In the limit $\mathrm{Re} \to \infty$ the continuous modes become undamped acoustic disturbances.

1. The stability of a plane-parallel flow of a viscous compressible gas to small two-dimensional traveling-wave perturbations of the form

$$\{\rho(y),\, u(y),\, v(y),\, p(y),\, \theta(y)\} \exp\,[i\alpha(x - ct)],\; c = c_r + ic_i \tag{1.1}$$

is determined by the well-known system of linear equations [6, 7]:

$$\begin{gathered}
D\rho + \sigma/T - T'v/T^2 = 0,\\
Du + U'v + i\alpha Tp = T[\mu_0(u'' - \alpha^2 u) + i\alpha\mu_0\sigma/3 +\\
+ T'(u' + i\alpha v) + (U'\theta)']/\mathrm{Re},\\
Dv + Tp' = T[\mu_0(v'' - \alpha^2 v) + \mu_0\sigma'/3 + 2T'(2v' - i\alpha u)/3 + i\alpha U'\theta]/\mathrm{Re},\\
D\theta + T'v + (\gamma - 1)T\sigma = \gamma T[\mu_0(\theta'' - \alpha^2\theta) + T''\theta +\\
+ 2T'\theta']/\mathrm{Re}\mathrm{Pr} + \gamma(\gamma - 1)\mathrm{M}^2 T[2\mu_0 U'(u' + i\alpha v) + U'^2\theta]/\mathrm{Re},\\
p = (\theta/T + T\rho)/\gamma\mathrm{M}^2,\ \mu_0 = T,\ D = i\alpha(U - c),\ \sigma = i\alpha u + v',\\
\mathrm{M} = U_1/(\gamma R T_1)^{1/2},\ \mathrm{Re} = \rho_1 U_1\delta/\mu_1.
\end{gathered} \tag{1.2}$$

Here x and y are the longitudinal and transverse coordinates; U(y) and T(y) are the velocity and temperature profiles of the unperturbed flow; ρ, u, v, p, θ are the amplitudes of the perturbations of the density, velocity along x and y, pressure, and temperature; α is the real wave number of the perturbation; c_r is the phase velocity; αc_i is the growth constant; Pr = 0.72 is the Prandtl number; $\gamma = 1.4$ is the adiabatic index; R is the gas constant. The x and y coordinates vary over $(-\infty, \infty)$ and they are made dimensionless with the help of the thickness of the shear layer δ. The other quantities are measured in terms of the unperturbed values in uniform flow at $y \to +\infty$ (denoted by the 1 subscript). A prime denotes differentiation with respect to y. It is assumed in (1.2) that the viscosity μ depends linearly on the temperature and that the pressure of the unperturbed flow is constant, in accordance with boundary-layer theory [8].

The boundary conditions for the perturbations express the requirement that their amplitudes must be finite at infinity:

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$$|\rho|,\ |u|,\ |v|,\ |p|,\ |\theta| < \infty \text{ when } y \to \pm\infty, \tag{1.3}$$

i.e., we include in the treatment perturbations whose amplitudes are finite at large distances from the shear layer. The linear equations (1.2) and the boundary conditions (1.3) form an eigenvalue problem for the unknown quantity $c = c_r + ic_i$. The flow will be unstable if $c_i > 0$, stable if $c_i < 0$, and neutrally stable when $c_i = 0$.

We assume that mixing of the parallel streams begins at the point x = 0, y = 0. In the region x > 0 the velocity and temperature profiles are self-similar solutions of the compressible boundary-layer equations with the boundary conditions

$$U \to 1,\ T \to 1 \text{ when } y \to \infty,\ U \to m,\ T \to \varkappa \text{ when } y \to -\infty, \tag{1.4}$$

where $m = U_2/U_1$, $\kappa = T_2/T_1$ are the velocity and temperature ratios of the streams.† Defining the thickness of the shear layer as $\delta = (\pi\mu_1 L/\rho_1 U_1)^{1/2}$, the solution has the form [4, 8]

$$U = 1 + \frac{1}{2}(m-1)\left[1 - \operatorname{erf}\left(\frac{1}{2}\sqrt{\pi}\eta\right)\right],$$

$$T = 1 + \frac{1}{2}(\varkappa - 1)\left[1 - \operatorname{erf}\left(\frac{1}{2}\sqrt{\pi \Pr}\eta\right)\right] + \frac{\Pr(\gamma-1)\mathrm{M}^2(m-1)^2}{4\sqrt{2-\Pr}}\int_\eta^\infty \Phi(z)\,dz, \tag{1.5}$$

$$y = \int_0^\eta T(z)\,dz,\quad \Phi(z) = \exp\left(-\frac{1}{4}\pi \Pr z^2\right)\operatorname{erf}\left(\frac{1}{2}\sqrt{\pi(2-\Pr)}\,z\right).$$

Here η is the Dorodnitsyn–Stewartson variable and L is the distance between the origin and a certain fixed cross section. Because $\mathrm{Re} = \pi L/\delta$, by varying Re in (1.2) we can study the stability of the flow in different cross sections downstream from the point where mixing begins (the quasiparallel approximation).

Calculations using (1.5) show that for $0 \le M \le 2$, $0.2 \le \kappa \le 5$ the velocity profile U(y) in the shear layer varies only slightly with M and κ. But the temperature profile is sensitive to the temperature difference between the streams and viscous dissipation, which is proportional to $M^2(\partial U/\partial y)^2$. The latter effect causes the form of the profile to change significantly at large M: The temperature in the center of the shear layer becomes higher than at the edges.

Before solving the stability problem (1.2)-(1.5), we discuss some useful transformations to reduce the computational labor. We see from (1.2)-(1.5) that the eigenvalue depends on m, κ, M, Re, α: $c = c(m, \kappa, M, \mathrm{Re}, \alpha)$. Analysis of the stability simplifies, however, if we note that the values of c for different values of m are very simply related to one another. We consider, for example, how the eigenvalues and eigenfunctions transform when we go from arbitrary m to m = 0 (the latter corresponds to the usual experimental situation: a boundary layer of a jet ejected into air at rest). We denote the eigenvalue and eigenfunction for m = 0 with a zero superscript. From (1.2)-(1.5)

$$c^0 = c(0, \varkappa, \mathrm{M}, \mathrm{Re}, \alpha) = \frac{\left[c\left(m, \varkappa, \frac{\mathrm{M}}{1-m}, \frac{\mathrm{Re}}{1-m}, \alpha\right) - m\right]}{1-m}, \tag{1.6}$$
$$\{\rho^0,\ u^0,\ v^0,\ p^0,\ \theta^0\} = \{\rho,\ (1-m)u,\ (1-m)v,\ (1-m)^2 p,\ \theta\}.$$

Physically (1.6) corresponds to transformation to a moving (with velocity m) reference frame and to scaling the velocity by the factor (1 − m).

Another relation connects the stability characteristics for temperature differences κ and $\tilde{\kappa} = 1/\kappa$:

$$\tilde{c} = c(m, \tilde{\varkappa}, \mathrm{M}, \mathrm{Re}, \alpha) = (1+m) - c^*(m, \varkappa, \mathrm{M}\sqrt{\varkappa}, \varkappa\mathrm{Re}, \alpha/\varkappa), \tag{1.7}$$
$$\{\tilde{\rho}(y), \tilde{u}(y), \tilde{v}(y), \tilde{p}(y), \tilde{\theta}(y)\} = \{\varkappa\rho^*(\tilde{y}), -u^*(\tilde{y}), -v^*(\tilde{y}),$$
$$\varkappa p^*(\tilde{y}), \theta^*(\tilde{y})/\varkappa\},\ \tilde{y} = -\varkappa y,$$

†$0 < \kappa < \infty$. We assume $-1 \le m \le 1$. This can always be arranged by choosing x in the direction of the stream with the higher velocity and the y axis from the shear layer toward the same stream.

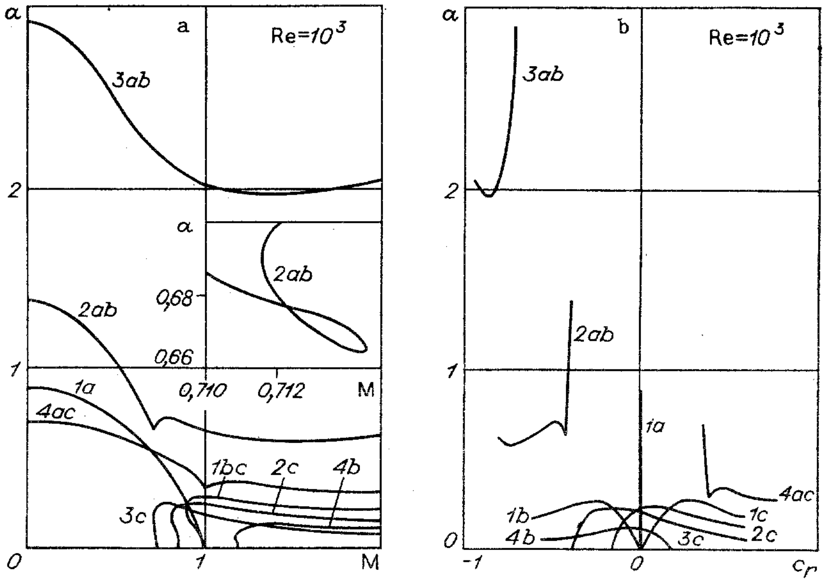

Fig. 1

where the star denotes the complex conjugate. Indeed, substituting (1.7) into (1.2) and taking the complex conjugate, we obtain the original system (1.2).

2. The stability calculations for the discrete eigenvalues are presented in Figs. 1-3 for a given temperature difference κ. The numerical method used to solve (1.2) and (1.3) is the same as in [4] and will not be described here. In all calculations m = −1; results for other values of m (in particular m = 0) can be obtained with the help of the transformation (1.6). We note that κ < 1 corresponds to a hot jet, i.e., the stream with the higher velocity is hotter, and κ > 1 corresponds to a jet colder than the surrounding air.

Neutral stability curves are drawn in the α, M and α, c_r planes in Figs. 1a and 1b for Re = 10^3, i.e., practically the inviscid limit. Curves 1-4 correspond to κ = 1, 0.5, 0.2, 2. The unstable regions in the α, M plane lie between the neutral curves and the α = 0 axis. The letters a, b, c denote neutral waves of different types (see Appendix). For example, the neutral curve 1 (κ = 1) consists of the segments 1a, 1b, and 1c. The segment 1a corresponds to neutral subsonic waves with c_r = 0. The segments 1b and 1c are neutral curves of the two supersonic modes. They coincide in the α, M plane (the curve 1bc) and the velocities of the modes are equal in magnitude (to c_r) and opposite in sign. This is a consequence of the symmetry properties of U(y) and T(y) at κ = 1 [4]. When κ = 1 all segments of the neutral curve join at the point α = 0, M = 1, c_r = 0. In this case the perturbation travels at the speed of sound with respect to both of the streams, i.e., $M_s|_{\kappa=1} = 1$ (see Appendix). Near the joining point the neutral curve 1 forms a loop in the α, M plane and both subsonic and supersonic neutral oscillations exist in a narrow region of Mach number ($M_* < M < 1$). Here M_* is the minimum Mach number for neutral supersonic perturbations ($M_*|_{\kappa=1} = 0.906$).

When κ ≠ 1 the neutral curve of the subsonic mode joins with one of the two neutral curves of the supersonic perturbations, forming a single continuous curve (2ab for κ = 0.5,

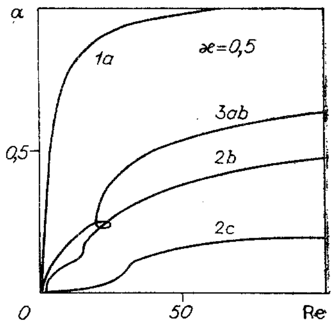

Fig. 2

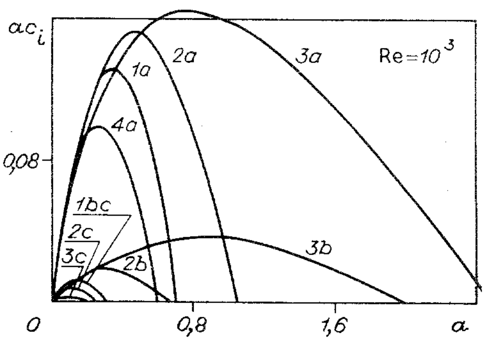

Fig. 3

TABLE 1

Perturbation mode	$\varkappa$				
	0,2	0,5	1	2	5
	M_*				
b	0,58	0,71	0,91	1,15	1,61
c	0,72	0,82	0,91	1,01	1,29
	α_{max}				
a	2,91	1,39	0,92	0,70	0,58
b	2,34	0,73	0,31	0,14	0,05
c	0,26	0,28	0,31	0,37	0,47

3ab for $\kappa = 0.2$, 4ac for $\kappa = 2$). The joining point no longer lies on the $\alpha = 0$ axis. The neighborhood of this point is shown in magnified scale in the α, M plane for $\kappa = 0.5$ in the insert of Fig. 1a. We see that as before the neutral curve forms a loop, although it is much smaller in size. For still smaller κ the loop disappears and the regions where subsonic and supersonic neutral perturbations exist no longer overlap (see the curves for $\kappa = 0.2$). It follows from Fig. 1b that the phase velocity of the neutral waves is nearly constant as long as the perturbation is subsonic (the nearly vertical lines in the figure) and $|c_r|$ of the supersonic perturbations of type b ($\kappa = 0.5$; 0.2) or c ($\kappa = 2$) increase rapidly beyond the point where the segments join.

The second supersonic mode forms a separate neutral curve (2c, 3c, 4b) when $\kappa \neq 1$ with a narrower unstable region in the α, M plane. The propagation velocity of neutral perturbations of this mode can change sign and the corresponding curves in Fig. 1b intersect the $c_r = 0$ axis. We note that this mode was omitted in [9], which also considered the stability of a viscous compressible shear layer. In [9] only the growth constant was calculated; the neutral curves were not constructed and therefore an overall picture of the stability was not obtained.

The effect of viscosity on the stability of the shear layer is shown in Fig. 2, where the neutral curves are given in the α, Re plane for $\kappa = 0.5$ and different values of M (curves 1-3 correspond to M = 0.5, 1.2, 0.8). It is evident from Fig. 2 that the neutral curve of the subsonic perturbations passes through the origin and hence the critical Reynolds number is $Re_c = 0$. But Re_c is nonzero for the supersonic waves. Therefore, if $M > M_s$ we always have $Re_c \neq 0$. When $M_* < M < M_s$ the neutral perturbation which was supersonic in the limit $Re \to \infty$ can become subsonic for finite Re if $|c_r|$ also decreases with decreasing Re. This situation is illustrated by curve 3ab ($\kappa = 0.5$, M = 0.8) with $Re_c = 0$, since the perturbation is subsonic for small Re. The calculation shows that the loop in the curve 3ab is near the point where the velocity of the neutral wave with respect to the gas stream is equal to the speed of sound. A loop in this case is not surprising, since the neutral curves in the α, M and α, Re planes are projections onto these planes of the neutral surfaces $c_i(\alpha, M, Re) = 0$, which are obviously self-intersecting when $\kappa = 0.5$. Neutral curves in the α, Re plane are not shown for other values of κ in the interest of brevity. As before, $Re_c = 0$ for subsonic perturbations and $Re_c \neq 0$ for supersonic perturbations. Table 1 gives the dependence of κ on M_* and α_{max}, where α_{max} is the maximum (for all M) wave number of the unstable perturbations of the given mode for $Re = 10^3$. Obviously, M_* decreases monotonically with decreasing κ and so in a hot jet unstable supersonic waves occur for smaller M. Since $Re_c \to \infty$ when $M \to M_*$ (see Fig. 4 in [4]), it follows from the monotonic dependence $M_*(\kappa)$ that for a given M there exists a $\kappa = \kappa_*$ such that $Re_c \to \infty$ when $\kappa \to \kappa_*$. If $\kappa \approx \kappa_*$ then Re_c is large and the viscosity has a significant effect on the stability of the shear layer.

It follows from the dependence of α_{max} on κ that the temperature difference affects the sizes of the unstable regions of the subsonic and supersonic perturbations differently. The unstable region of the subsonic mode expands with decreasing κ and contracts with increasing κ. The unstable region of the supersonic perturbations reaches a minimum size at $\kappa = 1$ and in this case heating and cooling destabilize the flow.

The effect of the temperature difference on the stability of the perturbations is shown in Fig. 3, where the growth constant αc_i is given as a function of α for $Re = 10^3$. Curves 1-4 corresponds to $\kappa = 1$, 0.5, 0.2, 2 and the different perturbation modes are labeled a, b, and c. The growth constant αc_i was calculated for the subsonic (supersonic) perturbations

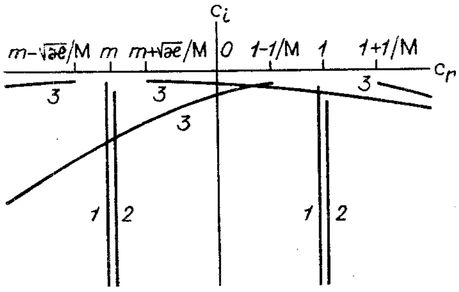

Fig. 4

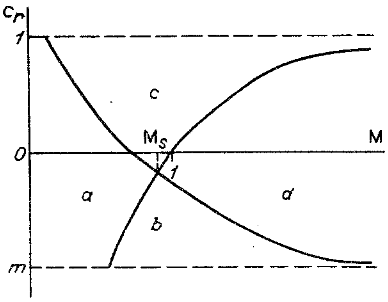

Fig. 5

for M = 0.5 (M = 1.2). It is evident that the growth constant for supersonic perturbations is much smaller (by about an order of magnitude) than for subsonic perturbations for all κ. We note also that an expansion of the region of unstable α is accompanied by an increase in the maximum value of αc_i.

3. It is well known [10] that for flow in free space the stability problem has a continuous spectrum, in addition to discrete eigenvalues. The characteristic feature of the perturbations of the continuous spectrum is the fact that their eigenfunctions do not vanish at infinity [10]. Therefore, to find the continuous spectrum we consider (1.2) in the limits $y \to \pm\infty$, where the velocity U and temperature T are constants. The general solution of (1.2) in the limit $y \to +\infty$ can be written as a superposition of six fundamental solutions propotional to $e^{\pm\beta_n y}$ (n = 1, 2, 3), where the β_n^2 are [4]:

$$\beta_n^2 = \alpha^2 + \Delta_n, \quad n = 1, 2, 3, \quad \Delta_1 = \mathrm{Re}\, D/T^2,$$

$$\Delta_{2,3} = \frac{\mathrm{Re}\,\mathrm{Pr}\,D}{2T^2} \frac{1 + q\left(\frac{4}{3} + \frac{\gamma}{\mathrm{Pr}}\right)}{1 + \frac{4}{3}\gamma q} \left\{1 \pm \sqrt{1 - \frac{4q}{\mathrm{Pr}} \frac{1 + \frac{4}{3}\gamma q}{\left[1 + q\left(\frac{4}{3} + \frac{\gamma}{\mathrm{Pr}}\right)\right]^2}}\right\}, \tag{3.1}$$

$$q = \frac{\mathrm{M}^2 DT}{\mathrm{Re}}, \quad D = i\alpha(U - c).$$

Here U = 1 and T = 1. The general solution in the limit $y \to -\infty$ is constructed in the same way, only U = m, T = κ in (3.1). The fundamental solutions corresponds to the different types of elementary disturbances in a compressible gas; at large Re they can be identified with vorticity, entropy, and pressure waves [4, 11].

According to the boundary conditions (1.3), the perturbation amplitude in a free stream decreases exponentially, except when one of the β_n^2 is real and negative:

$$\beta_n^2(c) = [(\beta_n)_r + i(\beta_n)_i]^2 = -k^2 \tag{3.2}$$

(k is a real number, $k \neq 0$). The asymptotic eigenfunction of the damped perturbation is a linear combination of the three fundamental solutions for which $(\beta_n)_r y < 0$.

It can be shown that the solution of the problem (1.2), (1.3) can be found for any c satisfying (3.2) either in the limit $y \to +\infty$ or in the limit $y \to -\infty$, i.e., a continuous spectrum exists in the problem. Indeed, in this case both solutions of the form $e^{\pm iky}$ satisfy (1.3). The asymptotic eigenfunction in one of the free streams will then be a superposition of four linearly independent solutions.

Writing (1.2) as a system of six first-order equations and integrating y = +∞ and y = −∞ to y = 0 (see the detailed description of the procedure for the numerical solution in [4]), we obtain the following matching condition at y = 0:

$$C_1\mathbf{z}_1 + C_2\mathbf{z}_2 + C_3\mathbf{z}_3 + C_4\mathbf{z}_4 = C_5\mathbf{z}_5 + C_6\mathbf{z}_6 + C_7\mathbf{z}_7 \tag{3.3}$$

($\mathbf{z}_1, \ldots, \mathbf{z}_7$ are the fundamental solution vectors at the point y = 0 and $C_1, \ldots, C_7$ are unknown constants to be determined). One of the constants can always be chosen arbitrarily and corresponds to normalizing the solution. Then (3.3) becomes a system of six linear inhomogeneous equations uniquely determining the remaining six constants and the eigenfunction, $-\infty < y < \infty$ and undamped in one of the streams.

We obtain from (3.2) for n = 1, 2, 3 and Re ≫ 1

$$c = U - \frac{iT^2(\alpha^2+k^2)}{\alpha\,\mathrm{Re}}; \tag{3.4}$$

$$c = U - \frac{iT^2(\alpha^2+k^2)}{\alpha\,\mathrm{Re}\,\mathrm{Pr}} + O(\mathrm{Re}^{-2}); \tag{3.5}$$

$$c = U \pm \frac{\sqrt{T(\alpha^2+k^2)}}{\alpha \mathrm{M}} - \frac{1}{2}\left(\frac{4}{3} + \frac{\gamma-1}{\mathrm{Pr}}\right)\frac{iT^2(\alpha^2+k^2)}{\alpha\,\mathrm{Re}} + O(\mathrm{Re}^{-2}). \tag{3.6}$$

The eigenvalues of the continuous spectrum for different k form lines in the complex plane $c_r + ic_i$ (see Fig. 4). Lines 1 and 2 correspond to the vortex and entropy branches of continuous spectrum (3.4) and (3.5). Each of these lines is composed of two members corresponding to waves with phase velocity $c_r = 1$ (undamped in the upper stream when $y \to \infty$, $U = 1$, $T = 1$) and to waves with $c_r = m$ (undamped in the lower stream with $y \to -\infty$, $U = m$, $T = \kappa$). It follows that these perturbations are carried along by the streams and their phase velocities relative to the gas are zero. The four segments of the acoustic branch of the continuous spectrum (3.6) (labeled 3) are pressure waves propagating with a velocity relative to the gas greater than or equal to the speed of sound $\sqrt{T}/M$. This branch is absent in the limit of an incompressible fluid. Indeed, when $M \to 0$ it follows from (3.6) that the velocity of acoustic perturbations $c_r \to \pm\infty$, which is consistent with an infinite speed of sound in an incompressible medium. We note that acoustic perturbations of the continuous spectrum can play an important role in problems of sound generation by supersonic shear flow [3] and in the opposite case of excitation of instability waves by an external acoustic perturbation [12].

At finite Re all perturbations of the continuous spectrum are stable and damp out in time. It is evident from (3.4)-(3.6) that the continuous spectrum becomes neutral ($c_i = 0$) only in the limit $\mathrm{Re} \to \infty$.

The supersonic discrete perturbations considered in Sec. 2 are closely related to acoustic waves of the continuous spectrum. Although in the presence of viscosity the discrete eigenfunctions always damp out as we move away from the shear layer, the damping can be very slow for large Re (see Figs. 5 and 6 in [4]). Indeed, expanding (3.1) for $\beta_3 = \beta_{3r} + i\beta_{3i}$ in powers of Re^{-1} we find that $|\beta_{3r}| \sim \mathrm{Re}^{-1}$. When $\mathrm{Re} \to \infty$ (1.2) reduces to a single second-order equation for the pressure amplitude [6, 7], whose solution is a pressure wave with a free-stream damping constant:

$$\beta = \lim_{\mathrm{Re}\to\infty} \beta_3 = \pm\alpha\sqrt{1 - \mathrm{M}^2(U-c)^2/T}. \tag{3.7}$$

For neutral waves of types b and c the damping constant becomes purely imaginary: $\beta = i\beta_i$. Far from the shear layer these waves have the form

$$e^{i(\alpha x + \beta_i y - \alpha c_r t)}, \tag{3.8}$$

i.e., they do not damp out but oscillate in x and y with a constant amplitude. Therefore, in the inviscid problem neutral supersonic perturbations of the discrete spectrum do not exist (although discrete unstable perturbations with $c_i > 0$ do exist [13]), and all such perturbations belong to the continuous spectrum. To study neutral perturbations of types b and c in the inviscid theory [13, 14] a more or less arbitrary additional condition must be introduced in order to single out one of the points of the continuous spectrum. In [13] this additional condition was taken to be the requirement that neutral oscillations must form a boundary to unstable oscillations when $c_i \to 0$, while in [14] it was assumed that the eigenfunctions far from the shear layer must have the form of outgoing waves.

It follows from (3.8) that undamped acoustic perturbations are radiated into the external stream or else approach the shear layer from the outside at an angle $\Psi = \arctan(\beta_i/\alpha)$ to the flow. The b and c waves must be radiated into the "upper" ($y > 0$) and "lower" sides of the shear layer, respectively. Obviously, these waves are analogous to Mach waves radiated from the boundary layer of a jet. Radiation of Mach waves into the surrounding space and inside the jet itself has been observed experimentally for sufficiently large M [15].

Appendix. Perturbation waves in a shear layer between two streams are naturally divided into four types (a, b, c, d) depending on the propagation velocity [9]. They correspond to four parts of the M, c_r plane (Fig. 5):

$$c_r = 1 - 1/M; \quad (A.1)$$

$$c_r = m + \sqrt{\varkappa}/M. \quad (A.2)$$

These equations describe perturbations propagating with respect to the upper (lower) stream with a velocity equal to the speed of sound in the given stream. Region *a* includes perturbations which are subsonic with respect to both streams. It follows from Fig. 5 that for this region there exists a maximum Mach number M_s corresponding to the intersection point of (A.1) and (A.2): $M_s = (1 + \sqrt{\kappa})/(1 - m)$.

It is well known [6] that the velocity of neutral perturbations c_N in region *a* is, for large Re, determined by the position of the generalized inflection point:

$$c_N = U(y_c), \quad \frac{d}{dy}\left(\frac{1}{T}\frac{dU}{dy}\right)\Big|_{y=y_c} = 0.$$

Perturbations in region b are supersonic with respect to the upper (y > 0) stream, perturbations in region c are supersonic with respect to the lower stream, and perturbations in region d are supersonic with respect to both streams. We note that perturbations of type d were not found here or in other papers known to us.

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